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Acoustic radiation force on coated cylinders in plane progressive waves

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Abstract

The purpose of this paper is to present the theory of the static acoustic radiation force on layered cylinders. The frequency dependence of the acoustic radiation force function Y_p (which is the radiation force per unit energy density and unit cross-sectional surface) for coated cylinders suspended in an incident plane wave sound field is analyzed, in relation to the thickness of the outer covering layer and the surrounding fluid. Explicit numerical calculations are presented for layered lossless cylinders immersed in water. The fluid-loading effect on the radiation force function curves is also analyzed by considering a high-density fluid surrounding the cylinders. These results show how absorption and the exterior fluid surrounding the cylinders affect the acoustic radiation force. It is shown here that the theory developed is much broader in scope compared to other existing theories.

1. Introduction

It is well known that radiation can exert a force [1]. The solar wind, for example, is caused by sunlight blowing away microparticles and its force on the surface of the sea is in the micro-Newton range. Similarly, sound waves exert a force (generally much larger than its electromagnetic counterpart) on objects placed along their path [2]. This force is called acoustic radiation force and is found to be useful in many applications [3].

The theory of acoustic radiation force was first investigated by Rayleigh [2]. Since then, a large amount of theoretical and experimental work has been performed on the acoustic radiation force acting on spheres. A detailed list of articles dealing with the acoustic radiation force can be found in Ref. [4]. The first theoretical study on cylinders goes back to the early work of Awatani [5] who developed an exact formula for the acoustic radiation force caused by progressive and stationary plane continuous-wave fields impinging on a rigid movable cylinder of variable density immersed in a compressible ideal fluid. In that study, the radiation force was numerically evaluated for a small range of size parameter values ($0 \leqslant ka \leqslant 5$). On the other hand, Zhuk [6] derived a long-wavelength approximation for the radiation force of progressive waves incident on a rigid

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movable cylinder. He computed the force for cylinders of variable densities immersed in non-viscous water in the range of $0 \leqslant ka \leqslant 2$. He showed that the radiation force increases with increasing frequency and significantly depends on the ratio of densitites of the fluid and cylinder. The theory of acoustic radiation force acting on a rigid movable cylinder in a progressive wave-field was extended to include its elasticity [7]. An approximate analytical solution (for $ka \ll 1$) for the acoustic radiation force exerted on a rigid movable cylinder was developed [8] and theoretical results were compared with experimental data obtained on a thin-walled microneedle. The agreement between theoretical and experimental results was about 20%. Further investigations were performed to study the acoustic radiation force on elastic shells in progressive waves [4], fluid compressible cylinders in stationary waves, to analyze the dynamics of long liquid columns [9], and rigid immovable, elastic, and viscoelastic cylinders in progressive [7,10] and stationary [11] plane waves. Lately, experimental and theoretical calculations of the radiation force on an elastic cylinder were obtained using the far-field derivation approach [12].

The purpose of this study is to extend the theory previously developed [7] to calculate the acoustic radiation force experienced by elastic cylinders coated by a sound absorbent layer and placed in a plane progressive continuous-wave field. Although a recent work dealing with the acoustic radiation force on coated cylinders in plane stationary waves has been published [13], a particularly important development and application of the theory for progressive waves is necessary in the field of non-destructive evaluation (NDE) for ultrasonic cleaning of layered cylinders immersed in non-viscous fluids [14]. Detection of delamination failure may benefit too, as repeated cyclic stresses, impact, etc., can cause layers to separate. Another application may consider estimating the covering layer thickness by inverting the problem arising from the radiation force function curves.

Here, analytical equation for the acoustic scattering of plane waves incident on a layered cylinder is derived and used to calculate the radiation force. Numerical calculations for the radiation force function Y_p—which is the radiation force per unit energy density and unit cross-sectional surface—are performed for a large range of size parameter values indicating how the acoustic radiation force function can be affected by variations of the cylinder's mechanical parameters, the thickness of the coating layer, and the surrounding fluid medium. The fluid-loading effect on radiation force was also examined by considering a high density fluid surrounding the cylinder (in this case mercury) chosen as an example.

2. Method

The acoustic radiation force experienced by a coated cylinder subjected to an incident continuous plane waves and immersed in an ideal fluid is determined by the solution to the linear scattering acoustic field disturbed by the presence of the layered cylinder. Hence, the acoustic scattering problem should be solved first. The radiation force is then determined by integrating the averaged radiation-stress tensor over the surface of the layered cylinder.

2.1. Acoustic scattering by the layered cylinder

The geometry and the coordinate system used are shown in Fig. 1. The center of the layered cylinder coincides with the origin of a rectangular coordinate system (x_0, y_0, z_0), and the plane waves approach the cylinder along the negative x_0-axis ($\theta = \pi$).

In the exterior fluid medium (medium 1), the linearized continuity and Euler's equations can be written as [15]

$$\frac{\partial \rho_1}{\partial t} + \rho_1 \nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho_1 \frac{\partial \mathbf{v}}{\partial t} + \nabla P = 0, \tag{2}$$

where ρ_1 is the mass density, P is the ambient pressure equal to P_0 in the absence of sound, and $\mathbf{v}$ is the fluid velocity. For an ideal (non-viscous) fluid, the linearized equation of state is $P = c_1^2 \rho'$, where c_1 is the speed of

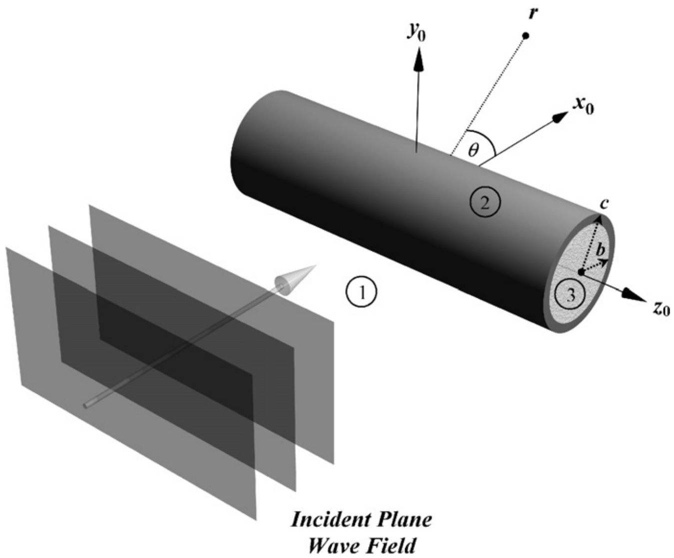

Fig. 1. A coated cylinder placed in a progressive plane-wave field incident in the direction $\theta = \pi$.

sound, and ρ' corresponds to the fluctuations of the medium density caused by the passage of the sound wave. Eqs. (1) and (2) can be combined to a single equation for the velocity v:

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} = c_1^2 \nabla(\nabla \cdot \mathbf{v}). \tag{3}$$

Assuming that the velocity v can be derived from a scalar potential φ_1, $\mathbf{v} = -\nabla\varphi_1$, Eq. (3) can be rewritten in an equivalent form:

$$\frac{\partial^2 \varphi_1}{\partial t^2} = c_1^2 \nabla^2 \varphi_1. \tag{4}$$

Assuming also that the incident field is composed of monochromatic plane waves, the solutions of Eq. (4) are of the form

$$\varphi_1(r, \theta, t) = \mathrm{Re}[\varphi_1(r, \theta, \omega)\,\mathrm{e}^{-\mathrm{i}\omega t}], \tag{5}$$

where Re[.] indicates the real part of a complex number, and $\varphi_1(r,\theta,\omega)$ may be complex. Replacing Eq. (5) in Eq. (4), and after some manipulation, the Helmholtz equation is obtained:

$$(\nabla^2 + k_1^2)\varphi_1 = 0, \tag{6}$$

where the compressional wavenumber in the fluid $k_1 = \omega/c_1$.

Therefore, the *total* scalar velocity potential field (solution of Eq. (6)) is the sum of the incident and scattered fields and it can be expressed in cylindrical coordinates by

$$\varphi_1(r, \theta) = \Phi_0 \sum_{n=0}^{\infty} \varepsilon_n i^n (J_n(k_1 r) + a_n H_n^{(1)}(k_1 r)) \cos(n\theta), \tag{7}$$

where Φ_0 is the amplitude, ε_n called the Neumann factor is defined as $\varepsilon_0 = 1$ and $\varepsilon_{n>0} = 2$, $J_n(\cdot)$ and $H_n^{(1)}(\cdot)$ are the cylindrical Bessel and Hankel functions of the first kind of order n, respectively, k_1 is the wavenumber in the exterior fluid medium (medium 1), and a_n are the unknown scattering coefficients that will be determined by the appropriate boundary conditions.

The waves inside the layered cylinder (media 2 and 3) will be represented by suitable solutions of the equation of motion of a solid elastic medium (since no absorption is included yet), which may be written as

$$(\lambda_{2,3} + \mu_{2,3})\nabla(\nabla \cdot \mathbf{U}_{2,3}) + \mu_{2,3}\nabla^2 \mathbf{U}_{2,3} = \rho_{2,3}\frac{\partial^2 \mathbf{U}_{2,3}}{\partial t^2}, \tag{8}$$

where $\lambda_{2,3}$ and $\mu_{2,3}$ are the Lamé coefficients, and $\rho_{2,3}$ the mass densities for the covering layer (medium 2) and core material (medium 3), respectively. $\mathbf{U}_{2,3}$ is the vector displacement that can be expressed as a sum of the gradient of a scalar potential $\Phi_{2,3}$ and the curl of a vector potential $\mathbf{\Psi}_{2,3}$ as follows:

$$\mathbf{U}_{2,3} = \nabla \Phi_{2,3} + (\nabla \times \mathbf{\Psi}_{2,3}). \tag{9}$$

Using the problem symmetry, the vector potential $\mathbf{\Psi}_{2,3}$ reduces to a scalar equation, i.e. $\mathbf{\Psi}_{2,3} = (0,0,\Psi_{2,3})$, and using the condition $\nabla \cdot \mathbf{\Psi}_{2,3} = 0$ (since $\mathbf{\Psi}_{2,3}$ is a solenoidal field), the Helmholtz equations for the solid medium are obtained as follows:

$$(\nabla^2 + k^2_{L,2,3})\Phi_{2,3} = 0, \tag{10}$$

$$(\nabla^2 + k^2_{S,2,3})\Psi_{2,3} = 0, \tag{11}$$

where $k_{L,2,3} = \omega/[(\lambda_{2,3} + 2\mu_{2,3})/\rho_{2,3}]^{1/2}$ and $k_{S,2,3} = \omega/[\mu_{2,3}/\rho_{2,3}]^{1/2}$, refer to the longitudinal and shear wavenumbers in the solid media, respectively.

The longitudinal and shear waves inside the layer (medium 2) are represented in cylindrical coordinates by

$$\Phi_2(r,\theta) = \Phi_0 \sum_{n=0}^{\infty} \varepsilon_n i^n (b_n J_n(k_{L,2}r) + c_n Y_n(k_{L,2}r)) \cos(n\theta), \tag{12}$$

$$\Psi_2(r,\theta) = \Phi_0 \sum_{n=0}^{\infty} \varepsilon_n i^n (d_n J_n(k_{S,2}r) + e_n Y_n(k_{S,2}r)) \sin(n\theta), \tag{13}$$

where $Y_n(\cdot)$ are the cylindrical Bessel functions of the second kind. Sound absorption by the polymer-type viscoelastic layer is modeled by introducing complex wavenumbers independent of frequency, accounting for losses inside the covering layer. Incorporating complex wavenumbers independent of frequency into the acoustic scattering theory holds only for *linear* viscoelasticity. Therefore the material's corresponding normalized absorption coefficients of compressional and shear waves are constant quantities [16,17].

In the core material (medium 3), the potentials solution of Eqs. (10) and (11) are given by

$$\Phi_3(r,\theta) = \Phi_0 \sum_{n=0}^{\infty} \varepsilon_n i^n f_n J_n(k_{L,3}r) \cos(n\theta), \tag{14}$$

$$\Psi_3(r,\theta) = \Phi_0 \sum_{n=0}^{\infty} \varepsilon_n i^n g_n J_n(k_{S,3}r) \sin(n\theta), \tag{15}$$

a_n, b_n, c_n, d_n, e_n, f_n, and g_n, are the unknown coefficients determined from the following boundary conditions:

- At the outside boundary of the coated cylinder (interface at medium 1 and 2), the displacements (velocities) and normal stresses must be continuous and the tangential stresses must be zero, leading to:
  - $v_r^{(1)}\big|_{r=c} = -\mathrm{i}\omega U_r^{(2)}\big|_{r=c}$;
  - $\sigma_{rr}^{(1)}\big|_{r=c} = \sigma_{rr}^{(2)}\big|_{r=c}$;
  - $\sigma_{r\theta}^{(2)}\big|_{r=c} = 0$.
- At the interface between the outer layer and core material (interface at medium 2 and 3), the radial and tangential displacements are continuous, and the radial and tangential stresses of adjoining materials are equal:
  - $U_r^{(2)}\big|_{r=b} = U_r^{(3)}\big|_{r=b}$;
  - $U_\theta^{(2)}\big|_{r=b} = U_\theta^{(3)}\big|_{r=b}$;
  - $\sigma_{rr}^{(2)}\big|_{r=b} = \sigma_{rr}^{(3)}\big|_{r=b}$;
  - $\sigma_{r\theta}^{(2)}\big|_{r=b} = \sigma_{r\theta}^{(3)}\big|_{r=b}$.

The general expressions of the velocities, displacements and stress components are similar to those obtained for the case of stationary waves [13]. The boundary conditions lead to seven linear equations with seven (scattering) coefficients. The general solution for a_n is given by

$$a_n = \frac{\begin{vmatrix} \lambda_1^* & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} & 0 & 0 \\ \lambda_2^* & \lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} & 0 & 0 \\ 0 & \lambda_{32} & \lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & \lambda_{44} & \lambda_{45} & \lambda_{46} & \lambda_{47} \\ 0 & \lambda_{52} & \lambda_{53} & \lambda_{54} & \lambda_{55} & \lambda_{56} & \lambda_{57} \\ 0 & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & \lambda_{66} & \lambda_{67} \\ 0 & \lambda_{72} & \lambda_{73} & \lambda_{74} & \lambda_{75} & \lambda_{76} & \lambda_{77} \end{vmatrix}}{\begin{vmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} & 0 & 0 \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} & 0 & 0 \\ 0 & \lambda_{32} & \lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & \lambda_{44} & \lambda_{45} & \lambda_{46} & \lambda_{47} \\ 0 & \lambda_{52} & \lambda_{53} & \lambda_{54} & \lambda_{55} & \lambda_{56} & \lambda_{57} \\ 0 & \lambda_{62} & \lambda_{63} & \lambda_{64} & \lambda_{65} & \lambda_{66} & \lambda_{67} \\ 0 & \lambda_{72} & \lambda_{73} & \lambda_{74} & \lambda_{75} & \lambda_{76} & \lambda_{77} \end{vmatrix}}, \tag{16}$$

where λ_1^*, λ_2^* and λ_{ij} are the dimensionless elements of the determinants given in Ref. [13].

2.2. Acoustic radiation force exerted on a layered cylinder

The acoustic radiation-stress tensor $\overline{\overline{\prod}}$ represents the transfer of momentum from the acoustic wave to the coated cylinder. This transfer of momentum results in the application of a force vector. The averaged force is defined as the integration of the time-average radiation-stress tensor [18] on the surface of the cylinder at rest S_0 (near-field), or on any closed surface S in the fluid that encloses the scatterer (far-field) as long as the cylinder is immersed in an ideal fluid. The averaged force is given by

$$<\mathbf{F}> = -\oint_S \left\langle \overline{\overline{\Pi}} \right\rangle \mathrm{d}\mathbf{S} = -\oint [<P - P_0> + \rho_1 <\mathbf{v}\cdot\mathbf{v}>]\mathrm{d}\mathbf{S}, \tag{17}$$

where $\mathrm{d}\mathbf{S} = \mathbf{n}\,\mathrm{d}S$($\mathbf{n}$ is the normal directed away from the cylinder) is the elementary area. The quantity $\langle P - P_0 \rangle$ is a "mean Eulerian excess pressure" [19] given up to second order by

$$\langle P - P_0 \rangle \simeq \rho_1 \left\langle \frac{\partial \varphi_1}{\partial t} \right\rangle + \frac{1}{2}\frac{\rho_1}{c_1^2}\left\langle \left(\frac{\partial \varphi_1}{\partial t}\right)^2 \right\rangle - \frac{1}{2}\rho_1 \left\langle |\nabla \varphi_1|^2 \right\rangle, \tag{18}$$

where ρ_1 and c_1 are the mass density and sound velocity in the exterior fluid medium (medium 1), respectively. Since the sound velocity potential is oscillatory, the time-average over a cycle is then $\langle \partial\varphi_1/\partial t \rangle = 0$. Thus, after inserting Eq. (18) into Eq. (17), the time-averaged force can be rewritten as [7]

$$\langle \mathbf{F} \rangle = -\iint_S \left[\left(\frac{1}{2}\frac{\rho_1}{c_1^2} \left\langle \left(\frac{\partial \varphi_1}{\partial t} \right)^2 \right\rangle - \frac{1}{2}\rho_1 \left\langle |\nabla \varphi_1|^2 \right\rangle \right) \mathbf{n} + \rho_1 \left\langle (v_n \mathbf{n} + v_t \mathbf{t}) v_n \right\rangle \right] \mathrm{d}S, \tag{19}$$

where $v_n\mathbf{n}$ and $v_t\mathbf{t}$ are the normal and tangential components of the fluid particle velocity of the boundary, respectively.

In the direction of wave propagation (x-direction) the value F_x of the total radiation force $\mathbf{F}$ is expressed as [7]

$$\langle F_x \rangle = \langle F_r \rangle + \langle F_\theta \rangle + \langle F_{r,\theta} \rangle + \langle F_t \rangle, \tag{20}$$

where

$$\begin{aligned}
\langle F_r \rangle &= \left\langle -\frac{1}{2} c\rho_1 \int_0^{2\pi} \left(\frac{\partial \varphi_1}{\partial r} \right)^2_{r=c} \cos\theta \, \mathrm{d}\theta \right\rangle, \\
\langle F_\theta \rangle &= \left\langle \frac{1}{2c} \rho_1 \int_0^{2\pi} \left(\frac{\partial \varphi_1}{\partial \theta} \right)^2_{r=c} \cos\theta \, \mathrm{d}\theta \right\rangle, \\
\langle F_{r,\theta} \rangle &= \left\langle \rho_1 \int_0^{2\pi} \left(\frac{\partial \varphi_1}{\partial r} \right)_{r=c} \left(\frac{\partial \varphi_1}{\partial \theta} \right)_{r=c} \sin\theta \, \mathrm{d}\theta \right\rangle, \\
\langle F_t \rangle &= \left\langle -\frac{1}{2c_1^2} c\rho_1 \int_0^{2\pi} \left(\frac{\partial \varphi_1}{\partial t} \right)^2_{r=c} \cos\theta \, \mathrm{d}\theta \right\rangle.
\end{aligned} \tag{21}$$

The final expression of the total force can be represented by

$$\langle F_x \rangle = \langle E \rangle S_c Y_p, \tag{22}$$

where $\langle E \rangle = \frac{1}{2}\rho_1 k_1^2 |\Phi_0|^2$ is the mean energy density of the incident plane acoustic wave field, and $S_c = 2c$ is the cross-sectional area for a *unit-length* cylinder. Y_p is a dimensionless factor called radiation force function that depends on the scattering and absorption properties of the cylinder target and is the radiation force per unit cross section and unit energy density.

After replacing the total velocity potential φ_1 (defined by Eq. (1)) in Eqs. (20) and (21) and manipulating the results, the expression of Y_p is greatly simplified and is given by

$$Y_p = -\frac{2}{k_1 c} \sum_{n=0}^{\infty} \left[\alpha_n + \alpha_{n+1} + 2(\alpha_n \alpha_{n+1} + \beta_n \beta_{n+1}) \right], \tag{23}$$

where α_n and β_n are real and imaginary parts of the scattering coefficients a_n defined by Eq. (16).

In a recent work [12], it was shown that Eq. (23) obtained from a near-field derivation approach is identical to the following equation:

$$Y_p^{\text{Far-field}} = -\frac{2}{k_1 c} \sum_{n=0}^{\infty} [\varepsilon_n \alpha_n + 2(\alpha_n \alpha_{n+1} + \beta_n \beta_{n+1})] \tag{24}$$

which is obtained from a far-field derivation approach. It has been recognized that calculations of radiation force based on far-field limits of the scattering are equivalent to those obtained from a near-field derivation in the idealized case of loss-less media. This result has been discussed for the case of cylinders placed in-plane stationary waves in an ideal fluid [13,20].

3. Numerical results and discussion

The calculations were performed for coated cylinders immersed in water and mercury, respectively, using Eq. (23). Both core and layer materials could be absorbent; however, in the following case, the outer layer consisted of phenolic polymer; a viscoelastic plastic material, and the core material of gold and stainless-steel materials, respectively, and they, let us note, are considered to be lossless. The mechanical parameters for these materials used in the calculations are given in Table 1. Absorption of sound inside the outer layer is included by introducing complex size parameters in the theory. Absorption in polymers was found to be of a hysteresis type (linearly dependent on frequency) so that the size parameters for both compressional and shear waves can be expressed in terms of frequency-independent factors [16].

Table 1
Material parameters used in the numerical calculations

Material	Mass density (10^3 kg/m^3)	Compressional velocity (m/s)	Shear velocity (m/s)	Normalized longitudinal absorption, γ_{21}	Normalized shear absorption, γ_{22}
Gold	19.3	3240	1200	—	—
Stainless steel	7.9	5240	2978	—	—
Phenolic polymer	1.22	2840	1320	0.0119	0.0257
Mercury	13.6	1407	—	—	—
Water	1.00	1500	—	—	—

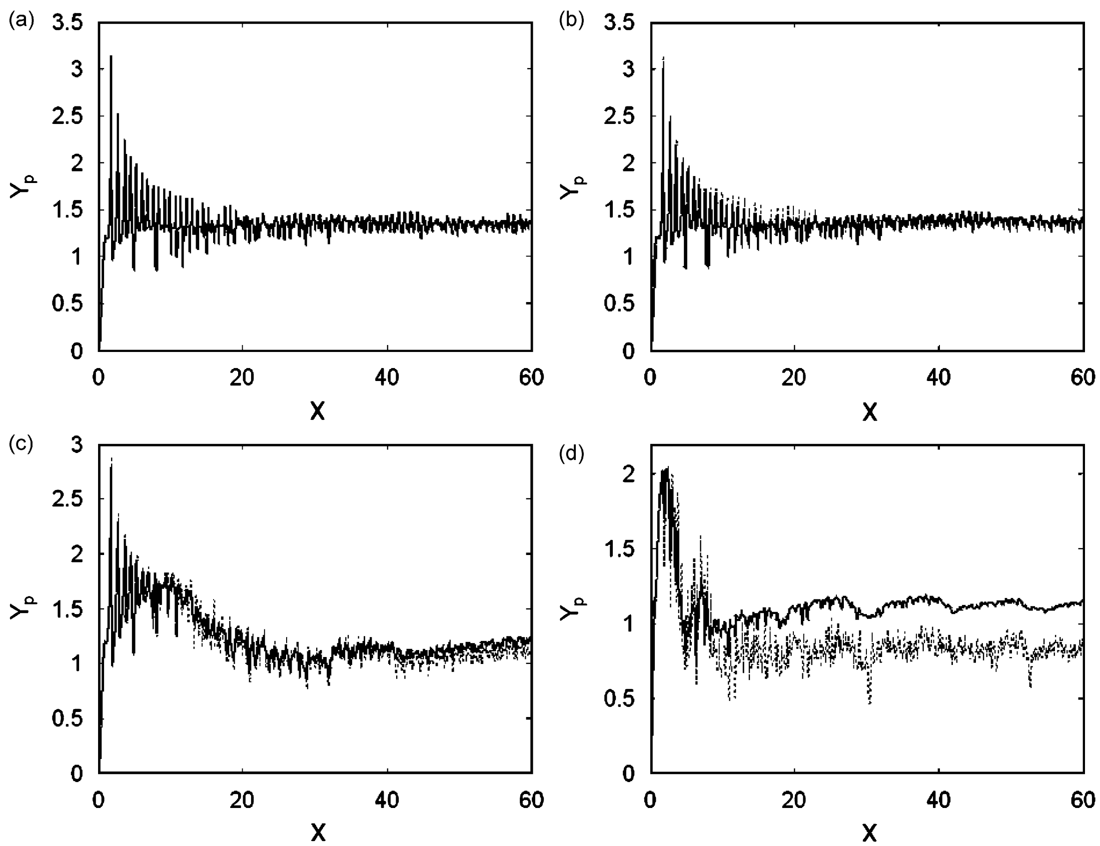

Fig. 2. The radiation force function Y_p versus k_1b for polyethylene-coated gold cylinders immersed in water for different thicknesses [(a) $e_1 = 1$; (b) $e_1 = 1.01$; (c) $e_1 = 1.1$; and (d) $e_1 = 1.5$] of the covering layer with (solid line) and without (dashed line) absorption.

The normalized absorption coefficients for both compressional and shear waves are listed in Table 1. The thickness of the viscoelastic layer is defined as the ratio of the outer radius to the inner radius of the layered cylinder $e_1 = c/b$ (Fig. 1).

Radiation force function curves were plotted as function of the size parameter $x = k_1b$ with particular emphasis on the effect of absorption, and the thickness of the outer covering by varying e_1. Computations for coated gold and stainless-steel cylinders were performed in a large range of size parameter values $0 \leqslant x \leqslant 60$ by intervals of 0.001. It is very important to choose a sufficiently small sampling step since resonance peaks are very sharp and a wrong sampling may lead to incorrect curves. It is also very important to extend the summation over the partial wave series to exceed the size parameter x to ensure proper convergence.

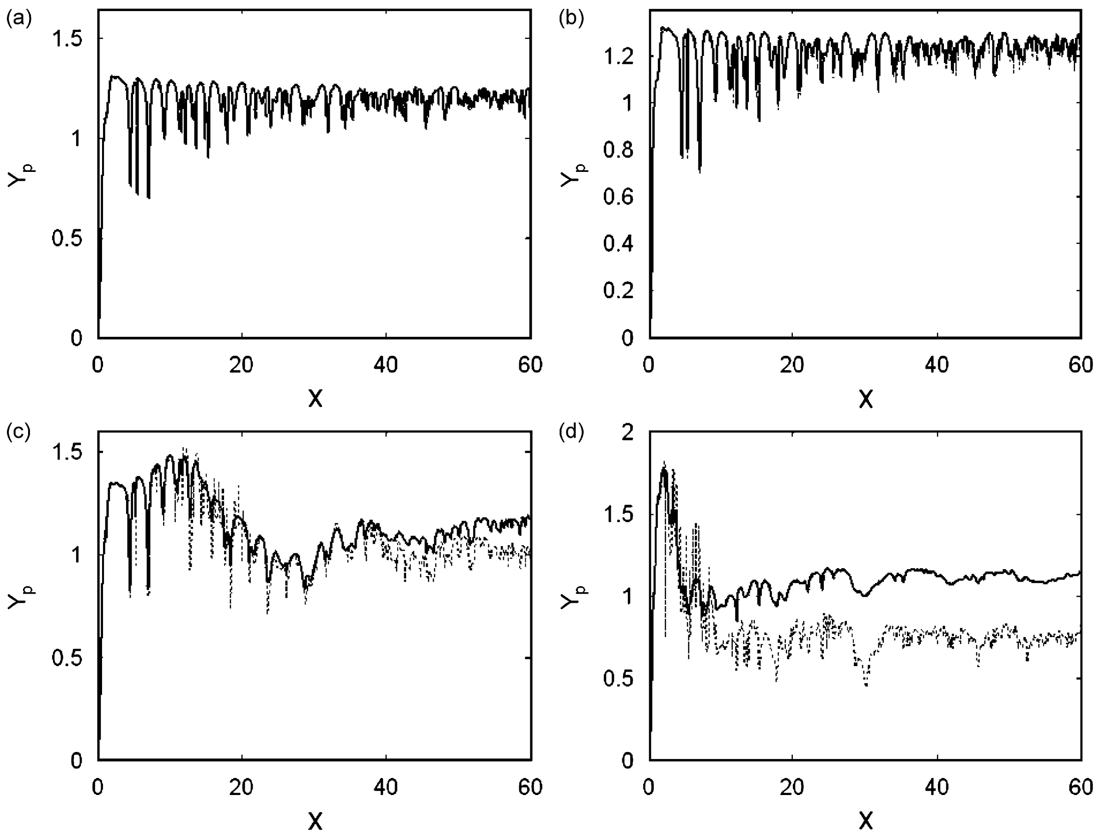

Fig. 3. The same as in Fig. 2 but for polyethylene-coated stainless-steel cylinders.

As initial test, the calculations for uncoated ($e_1 = 1$) gold (Fig. 2a) and stainless-steel (Fig. 3a) cylinders immersed in water were performed and compared to Figs. 3 and 5 of Ref. [7]. Excellent agreement was found.

Cases of coated cylinders immersed in water are shown in Figs. 2b–d and 3b–d for gold and stainless-steel materials, respectively, with and without absorption in the viscoelastic layer. Figures reveal that decrease in the amplitude of peaks is mainly due to absorption. Moreover, the effect of increasing the thickness of the outer covering (by varying e_1; $e_1 = 1;1.01;1.1;1.5$) (Figs. 2b–d and 3b–d) drastically changes radiation force.

Increase in the radiation force function amplitude values for thick absorptive layers, especially at high size parameter values (Figs. 2d and 3d) where damping of all peaks appears more clearly it is particularly noteworthy. This enhancement at high size parameter values in the radiation force function's amplitude is related to sound-energy absorption; when absorption is strong, the sound-energy density in the area of incident plane wave field is higher when compared to the case without absorption. Hence, the net force per cross-section acting on the viscoelastically coated cylinder in the direction of the incident waves is high.

Figs. 4a–d and 5a–d show additional calculations of the acoustic radiation force function curves for coated gold and stainless-steel cylinders immersed in mercury, with and without the inclusion of absorption within the viscoelastic layers. Obviously, the effect of fluid loading on coated cylinders is more prominent for the stainless-steel material whose mass density is relatively low with respect to the surrounding fluid. The fluid loading produces interactions between various resonance vibrational modes that can have significant effect on radiation force. This is clearly observed in Fig. 5d where the first resonance dip in case of water (Fig. 3) is transformed into a giant resonance peak at low size parameter values. A similar

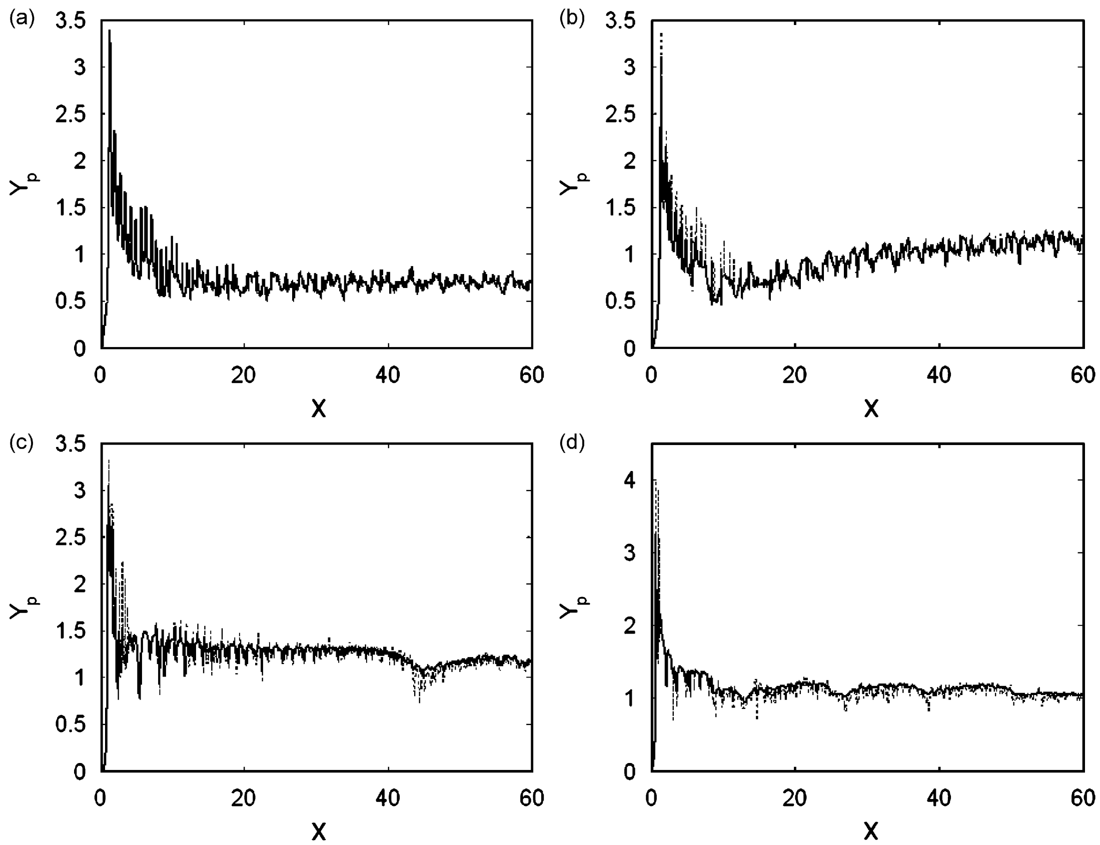

Fig. 4. The radiation force function Y_p versus k_1b for polyethylene-coated gold cylinders immersed in mercury for different thicknesses [(a) $e_1 = 1$; (b) $e_1 = 1.01$; (c) $e_1 = 1.1$; and (d) $e_1 = 1.5$] of the covering layer with (solid line) and without (dashed line) absorption.

behavior has been observed in acoustic backscattering from viscoelastic cylinders immersed in a high density fluid [21].

One notices also from Figs. 4a–d and 5a–d that, for the case of coated cylinders immersed in a high density fluid, the effect of absorption inside the viscoelastic layer is less pronounced at high size parameter values.

In the figures, the positions of minima and maxima are determined by the coated cylinder's material properties. A detailed discussion on whether the resonances are manifested as either maxima or minima in the Y_p curves is given in a previous publication [22].

4. Conclusion

In this work, the theory of the acoustic radiation force due to progressive incident plane waves of a continuous field impinging on coated cylinders immersed in inviscid fluids is examined. Basic calculations are performed for different materials and different thickness of the outer covering viscoelastic layers. The results of numerical calculations are presented indicating the ways in which the radiation force function curves are affected by variations of the cylinders' mechanical properties. The proposed model leads to an extension of the standard theory on the acoustic radiation force experienced by elastic cylinders since its corresponding results are obtained here by allowing $e_1 = 1$. The results for cylindrical shells can also be obtained by considering the core material as a fluid medium (shear velocity equal to zero) coated by an elastic (or a viscoelastic) layer.

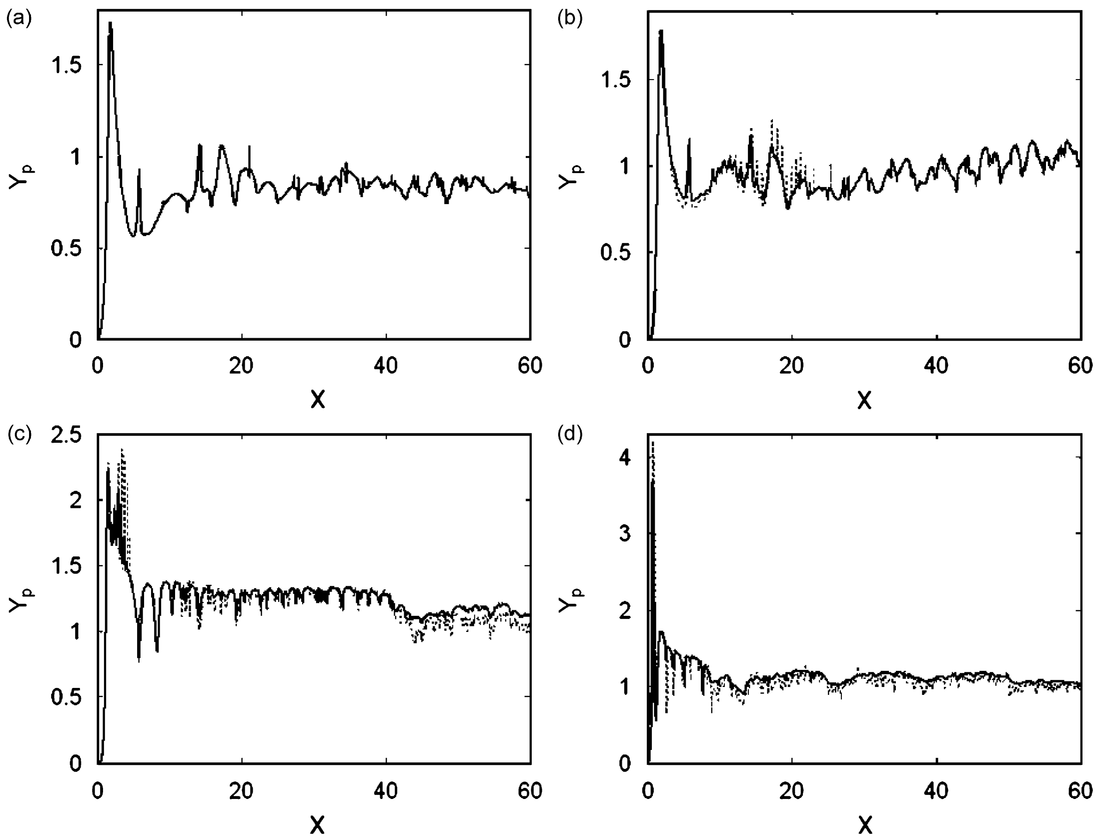

Fig. 5. The same as in Fig. 4 but for polyethylene-coated stainless-steel cylinders. One notices the high resonance peak at low size parameter values especially when the thickness of the viscoelastic layer increases (d).

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